1. (7 pts) Find the general solution of the following differential equation

$$\frac{dy}{dt} = 1 + \frac{1}{y^2}.$$

2. (13 pts) Find the solution of the initial-value problem using the method of undetermined coefficients

$$\begin{aligned} y'' + 2y' + 5y &= \sin t + e^{-t}, \\ y(0) &= 1, \\ y'(0) &= 1. \end{aligned}$$

3. (10 points) A body with the mass $250g$ is falling from rest towards the earth from the great height. As it falls the air resistance acts upon it and we shall assume that this resistance (in newtons) is numerically equal to $2v$, where v is the velocity (in meters per second). Find the velocity and distance fallen at time t seconds. Take $g = 10\ m/s^2$.

4. (10 pts) Find the general solution of the following linear differential equation. Use method of variation of parameters to find its particular solution.

$$y'' + 9y = 9\sec^2 3t, \quad 0 < t < \frac{\pi}{6},$$

note, that

$$\sec t = \frac{1}{\cos t},$$

and the particular solution must be found in the form

$$y_p = v_1(t)\, y_1(t) + v_2(t)\, y_2(t),$$

where $y_1(t), y_2(t)$ is the fundamental set of solutions for the corresponding homogeneous equation.

DO NOT use the formula, show ALL your steps.

5. (13 pts) Find the general solution of the following linear system of differential equations

$$\frac{dx}{dt} = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix} x.$$

Find the particular solution of the initial-value problem with the initial conditions

$$x(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

6. (13 pts) Use the Laplace transform to solve the following initial-value problem

$$y'' + 3y' + 2y = u_2(t), \quad y(0) = 0, \quad y'(0) = 1.$$

7. (7 points) Consider the system

$$\frac{dx}{dt} = \begin{pmatrix} 3 & 4 \\ 3 & 2 \end{pmatrix} x.$$

   (a) Find the eigenvalues and eigenvectors of the coefficient matrix.

   (b) Find the general solution of the system.

   (c) Classify the critical point $(0,0)$ according to its type.

8. (7 pts) For the given system of differential equations

$$\begin{aligned} \frac{dx}{dt} &= -x + y + 2xy, \\ \frac{dy}{dt} &= -4x - y + x^2 - y^2, \end{aligned}$$

   (a) verify that $(0,0)$ is a critical point,

   (b) show that the system is locally linear,

   (c) discuss the type and the stability of the critical point $(0,0)$ by examining the corresponding linear system.

9. (7 points) Consider the system

$$\begin{aligned} \frac{dx}{dt} &= -3x + x^3 + 2xy^2 \\ \frac{dy}{dt} &= -2y + \frac{2}{3}y^3 \end{aligned}$$

   Construct a Liapunov function of the form

$$ax^2 + by^2$$

   where a and b are constants and use the function to determine whether the critical point $(0,0)$ of the system is asymptotically stable or at least stable.

10. (10 pts) Determine all periodic solutions, all limit cycles, and the stability characteristics of all periodic solutions of the autonomous systems given in polar coordinates

$$\begin{aligned} \frac{dr}{dt} &= r\left(1 - r^2\right)\left(r^2 - 9\right), \\ \frac{d\theta}{dt} &= 1. \end{aligned}$$

PLEASE SHOW SUFFICIENT WORK IN YOUR SOLUTIONS
YOU CAN USE YOUR CALCULATORS FOR MATRIX OPERATIONS